Third Semester B.E. Degree Examination, December 2010
Advanced Mathematics – I

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions.*

1.   a.   Find the nth derivative of $\log(ax + b)$.   **(06 Marks)**

    b.   Find the nth derivative of $\dfrac{x}{\left(1+3x+2x^2\right)}$.   **(07 Marks)**

    c.   If $x = \sin t$ and $y = \cos mt$, prove that $(1 - x^2) y_{n+2} - (2n + 1) xy_{n+1} + (m^2 - n^2) y_n = 0$.   **(07 Marks)**

2.   a.   Show that the following pair of curves intersect each other orthogonally.
      $r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$.   **(06 Marks)**

    b.   Find the pedal equation of the curve $\dfrac{2a}{r} = 1 + \cos\theta$.   **(07 Marks)**

    c.   Find the first five terms of the Maclaurin series of $f(x) = \log \sec x$.   **(07 Marks)**

3.   a.   If $u = e^{ax - by} \sin(ax + by)$, show that $b\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y} = 2abu$.   **(06 Marks)**

    b.   If $u = \sqrt{x^2 + y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, find $\dfrac{du}{dx}$ when $x = y = a$.   **(07 Marks)**

    c.   If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that,
$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$
     (07 Marks)

4.   a.   Obtain the reduction formula for $\int \cos^n x\,dx$, where n is a positive integer.   **(06 Marks)**

    b.   Show that $\displaystyle\int_0^{\pi} \frac{\sqrt{1-\cos\theta}}{1+\cos\theta} \sin^2\theta \, d\theta = \frac{8\sqrt{2}}{3}$.   **(07 Marks)**

    c.   Evaluate $\displaystyle\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 y \, dy \, dx$.   **(07 Marks)**

5.   a.   Prove that $\left|\overline{\tfrac{1}{2}}\right| = \sqrt{\pi}$.   **(06 Marks)**

    b.   Show that $\displaystyle\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$.   **(07 Marks)**

    c.   Prove that $\beta(m,n) = \dfrac{\overline{|m}\ \overline{|n}}{\overline{|m+n}}$.   **(07 Marks)**

6.   a.   Solve $(e^4 + 1)\cos x \, dx + e^4 \sin x \, dy = 0$.   **(06 Marks)**

    b.   Solve $\left(x\tan\frac{y}{x} - y\sec^2\frac{y}{x}\right)ds + x\sec^2\left(\frac{y}{x}\right)dy = 0$.   **(07 Marks)**

    c.   Solve $(x + \tan y)\,dy = \sin 2y \, dx$.   **(07 Marks)**

7   a.   Solve $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{-2x}$.   (06 Marks)

   b.   Solve $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - 5y = \cos 3x$.   (07 Marks)

   c.   Solve $(D^2 - 5D + 1)y = 1 + x^2$.   (07 Marks)

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   a.   Prove that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1} \cos^n\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{n\theta}{2}\right)$.   (06 Marks)

   b.   Use Demoivre's theorem and solve the equation $x^4 - x^3 + x^2 + 1 = 0$.   (07 Marks)

   c.   Expand $\cos^8\theta$ in a series of cosine of multiples of θ.   (07 Marks)

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